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Fatigue curve parameters description of AISI 1141 Microalloyed Steel using Gatts equation, Exponential & Hyperbolic function.

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Abstract: Fatigue is observed to be one of the major reasons for failure of mechnical components subjected to cyclic load. Micro-alloying of steel is usually done in order to improve its fatigue strength. Micro-alloyed steel contains small quantity of alloying elements such as niobium, vanadium, molybdenum, zirconium, titanium, boron, and rare-earth metals. Motivation of the present work is to find the exact number of cycles to failure for a given stress. The fatigue curve can be described by Gatts equation, exponential function and hyperbolic function. The exactness of each method is checked by three stress level analysis. Gatts equation is a better tool to describe the fatigue curve at high levels of stresses which is near to ultimate stress.

Key Words: Endurance limit, Gatts equation, Microalloyed steel.

I. Introduction: Most important micro-alloying elements in steel are V, Ti, Nb and recently B. Boron exceptionally improves the hardenability of steel [1]. Micro-alloying facilitate obtaining of the required plastic properties in materials with concurrent preservation of strength properties. Notable progress has been made to improve the fatigue performance of the automotive components leading to improved component life and smaller power transfer system [2]. Bar steel for fatigue-sensitive automotive applications, such as gears and shafts have been developed. In automobiles and other engineered mechanical machines, large components are made of forged steels. Microalloyed forged steels used as a reasonable alternative to the quenched and tempered steels. Microalloyed steels have improved mechanical properties compared to quenched and tempered steels which originate from microstructural modification due to alloying elements such as vanadium (V), niobium (Nb), titanium (Ti) and aluminum (Al) [3].

Most of the service failures of metal components are caused by fatigue [4]. Fatigue is the process of progressive localized stable structural change taking place in a material subjected to conditions that create fluctuating stresses at some point or points and that may culminate in cracks. Fatigue is the most frequent cause of mechanical failure for components. Quenching and tempering are observed to lower the fatigue properties compared to pristine state. Tempering improves fatigue properties in a much better manner than quenching. Fatigure failure arrises due to heterogeneous stress distribution due to complex structural geometries and varying/fluctuating stresses.

At low loading levels, cracks nucleate in the weakest places inside the element [5]. Rolling contact fatigue (RCF) is the prime mode of failure in gears, which results from the repeated contact loads that fabricate alternating shear stresses in the contact surface. RCF results in disintegration of small bits of material from the surface. RCF potency is the key factor that decides the fitness of the material for application in gears. The RCF failure take place in three steps: (a) nucleation of crack from the surface, (b) propagation of crack, and (c) removal of material from the surface [6]. The factors that change RCF life are lubrication system, lubricant purity, surface topology, slip, residual stresses and material inclusion.

II.Gatts Equation: Gatts equation is [11]

$$K = \frac{1}{N}\left[\frac{1}{\sigma-\sigma_E} - \frac{1}{\sigma(1-C)}\right] \qquad (1)$$

Where, N is number of cycles to failure; σ is maximm cyclic stress Mpa; σ_E is endurance limit Mpa; K, (1-C) co-efficents of Gatts equation.

This equation relates applied stress and fatigue life for any material by using coefficients K and (1-C). Fatigue Test result of AISI 1141 microalloyed steel tabulated in Table (1).

Table (1): Fatigue test results of AISI 1141 microalloyed steel.

Stress level number	σ MPa	Number of cycles (*N*)
1	501	13982
2	401	57986
3	282	519680
4	255	1512906

Determination of Gatts coefficients and endurance limit by three stress level analysis: Take three stresses and its corresponding lives $\sigma_1, \sigma_2, \sigma_3$ are the stresses N_1, N_2, N_3 are its corresponding lives. The stress levels $\sigma_1, \sigma_2, \sigma_3$ are satisfy the condition $\sigma_1 \rangle \sigma_2 \rangle \sigma_3$. From the table (1) it is possible to make 24 combinations but 4 Stress combinations only satisfy the condition**.**

$$K_1 = \frac{1}{N_1}\left[\frac{1}{\sigma_1-\sigma_E} - \frac{1}{\sigma_1(1-C)}\right] \qquad (2)$$

$$K_2 = \frac{1}{N_2}\left[\frac{1}{\sigma_2-\sigma_E} - \frac{1}{\sigma_2(1-C)}\right] \qquad (3)$$

$$K_3 = \frac{1}{N_3}\left[\frac{1}{\sigma_3-\sigma_E} - \frac{1}{\sigma_3(1-C)}\right] \qquad (4)$$

Let us equate the right-hand members of the equations 2, 3, 4 to one another and find the calculated endurance limit, K,(1-C). The relative error % also calculated for four possible stress combinations [7, 8].

Table (2): Coefficents of Gatts equation (1-C), K and calculated endurance limit values at three stress level analysis with error % for AISI 1141 microalloyed steel.

Stress combination number *j*	Stress level number *i*	σ_E cal MPa	Gatts equation coefficients		Error %
			1-C	$K*10^{-8}$	
1	1,2,3	241.27	0.5958	3.5794	0.53
2	1,2,4	237.77	0.6007	3.4062	0.93
3	1,3,4	235.93	0.5960	3.0320	1.70
4	2,3,4	235.66	0.5763	2.9692	1.80

III. EXPONENTIAL FUNCTION: The fatigue curve of material is expressed in the form of exponential equation

$$\sigma = \sigma_E \exp\left(\frac{A}{N+B}\right) \qquad (5)$$

where σ = Maximum cyclic stress (MPa); σ_E = Endurance limit (MPa); N=Number of cycles to failure under the stress σ; A, B are exponential function parameters.

Three stress level analysis: To find the exponential function parameters A, B take three stress levels $\sigma_1, \sigma_2, \sigma_3$ and the corresponding number of cycles to failure N_1, N_2, N_3. There are 24 combination of three stresses possible, for four stresses 24 combination of three stresses survive but 4 combinations only suit the condition $\sigma_1 > \sigma_2 > \sigma_3$. The parameter B in the equation can be found from the relation

$$B = \frac{(N_3 - N_1)(N_2 ln\sigma_2 - N_1 ln\sigma_1) - (N_2 - N_1)(N_3 ln\sigma_3 - N_1 ln\sigma_1)}{(N_3 - N_1)(ln\sigma_1 - ln\sigma_2) - (N_2 - N_1)(ln\sigma_1 - ln\sigma_3)} (6)$$

From the value of B, value of A can be found from the following relation (7)

$$A = \frac{(N_1 + B)(N_2 + B)(ln\sigma_1 - ln\sigma_2)}{(N_2 - N_1)} (7)$$

The calculated fatigue limit σ_{Ec} calculated from the following expression (8)

$$ln\sigma_{Ec} = \ln\sigma_1 - \left(\frac{A}{N_1 + B}\right) = \ln\sigma_2 - \left(\frac{A}{N_2 + B}\right) \qquad (8)$$

Table (3): Parameters of exponential function for fatigue curve of AISI 1141 microalloyed steel.

Number of the combination of stresses *j*	Stress level number *i*	B	A	$\sigma_{endurance}$ calculated (σ_{Ec}), MPa	Δσ enduranc, MPa	ERROR %
1	1,2,3	75754.55	60722.92	254.65	14.65	6.10
2	1,2,4	84236.39	70677.47	243.96	3.96	1.65
3	1,3,4	132751.02	108795.71	238.68	1.31	0.54
4	2,3,4	164983.59	116406.82	237.90	1.10	0.87

Dynamic changes in exponential function parameters: To investigate the variation in exponential function parameters and endurance limit the stress level σ_1 & σ_2 and the corresponding number of cycles to failure N_1 & N_2 are taken in such a way that σ_1 is maximum stress and σ_2 is minimum stress for a given pair of stresses. It is possible to set the parameter B with the accuracy of 10^3. The Factor 'A' determined from equation (7).

Table (4): Dynamics of changes in the parameters of the exponential function.

B	A	σ_E endurance limit, MPa
120000	98571	240.06
121000	99368	239.95
122000	100166	239.85
123000	100964	239.74
124000	101763	239.63

Form table (4), it is perceive that when the value of B increases A also increases. When B and A increases the calculated endurance limit decreases. The most advantageous value of parameters A, B of the exponential function is accepted when the actual fatigue limit deviation doesn't exceed 1%. It is evident from the above table and the recommended value of B for microalloyed steel is 1,20,000 which has close association with experimental value.

Two stress level analysis: Take two stress levels σ_1, σ_2 and its corresponding number of cycles to failure N_1, N_2. The equation (5) can be written with respect A and B for two stress levels. The parameter B is determined by the formula

$$B = \frac{N_2(ln\sigma_2 - ln\,\sigma_R) - N_1(ln\sigma_1 - ln\sigma_R)}{(ln\sigma_1 - ln\sigma_2)} \quad (9)$$

From the relation (6) find the value of B and the parameter A is determined as follows.

$$A = (N_i + B)(ln\sigma_i - ln\sigma_R) \quad (10)$$

The accuracy of the relation (10) is checked by finding the value of A at two stress levels and equality of A is evident for the accuracy of equation (6) stress level combinations are made and the parameters A, B are tabulated in Table (5).

Table (5): Parameters of exponential function for fatigue curve of AISI 1141 microalloyed steel.

Number of the combination of stresses *j*	Stress level number *i*	B	A
1	1,2	87472.2	74666.98
2	1,3	127923.5	104437.80
3	1,4	120574.5	99029.13
4	2,3	153505.6	108563.40
5	2,4	136854.7	100016.10
6	3,4	78609.4	96485.02

From the table (5), it is observed that the values of B and A are indiscriminately varying.

IV. HYPERBOLIC FUNCTION

The hyperbolic relationship between fatigue life and applied stress is collectively with ultimate stress is given in equation (11)

$$N = \beta \frac{\sigma_u - \sigma}{\sigma - \sigma_E} \quad (11)$$

where σ_u = Ultimate stress of a material (MPa); σ_E = Endurance limit (MPa); σ = Maximum cyclic stress (MPa); β =co-efficient of hyperbolic function; N =Number of cycles to failure under the stress σ.

Two stress analysis: Take any two random stresses such that $\sigma_1>\sigma_2$ and N_1, N_2 are respective lives corresponding to σ_1, σ_2. To calculate the endurance limit of microalloyed steel the value of β found from equation (12). β is the hyperbolic coefficient which is numerically equal to number of cycles to failure under the stress σ_β. $\sigma_\beta=0.5(\sigma_u+\sigma_E)$.

$$\beta = \frac{(\sigma_1 - \sigma_2)N_1N_2}{(\sigma_u - \sigma_1)N_2 - (\sigma_u - \sigma_2)N_1} \quad (12)$$

From the value of β, σ_{EC} can calculated by equation (13) and its equality resultant to both stress levels tartan by equation (13).

$$\sigma_{Ec} = \sigma_1 - \frac{\beta}{N_1}(\sigma_u - \sigma_1) = \sigma_2 - \frac{\beta}{N_2}(\sigma_u - \sigma_2) \quad (13)$$

From table (1), 24 realistic pair of stresses can be arrived but 6 pair of stresses only satisfies the condition. For each pair of stress the value of β, σ_{EC} are calculated by equation (12) and (13). The % of deviation from original value also tabulated in table (6).

Table (6): Hyperbolic function Coefficient β , Calculated endurance limit calculated endurance limit σ_{EC}, Relative Error.

Stress combination number *j*	Stress level Number	β	Calculated endurance limit σ_{EC}.	% error
1	1,2	254991.34	210.24	13.30
2	1,3	95514.68	234.51	2.26
3	1,4	94581.87	237.18	1.17
4	2,3	62609.47	250.92	4.55
5	2,4	66100.70	242.55	1.06
6	3,4	87639.53	238.49	0.63

From table (6), it is observed that the deviation is higher when the stress combination consists of higher level and it attain minimum in lower level of stresses. The lowest divergence acknowledged in a pair of 3, 4. The value of β increases when the stress value decreases. The divergence is caused due to redundant use of the factor ultimate stress.

Three stress level analysis: The usage of ultimate stress is not yield a good result even it has direct proportion with endurance limit. So it is essential to find the factor limiting stress or stress limit σ_{lj} corresponding j^{th} combination of three stress level is determined by equation (14).

$$\sigma_{lj} = \frac{(\sigma_1 N_3 - \sigma_3 N_1)(\sigma_2 - \sigma_3)N_2 - (\sigma_2 N_3 - \sigma_3 N_2)(\sigma_1 - \sigma_3)N_1}{(N_3 - N_1)(\sigma_2 - \sigma_3)N_2 - (N_3 - N_2)(\sigma_1 - \sigma_3)N_1} \quad (14)$$

The stress limit is taken as equivalent to ultimate stress and coefficient β, calculated endurance limit σ_{EC} found by equation (12) and (13).

Table (7): Limiting stress, coefficient of hyperbolic function β, Calculated endurance limit σ_{EC}, and Error %.

Sr. No.	Stress level number	stress limit σ_{lj}	Co-efficient β	calculated endurance limit σ_{EC}	% error
1	1,2,3	569.10	50711.84	253.98	5.83
2	1,2,4	566.11	55264.73	243.64	1.51
3	1,3,4	543.47	86409.51	238.52	0.62
4	2,3,4	484.34	113554.40	237.78	0.92

From table (7), the usage of limiting stress is not making a radical change in accurateness of endurance limit calculation. But it reduces the altitude of error. The impact of limiting stress is insignificant because it shows an indiscriminate error in endurance limit for very closer values. Here the combination of stresses plays vital role and its selection direct to the accuracy of solution.

Stress limit and hyperbolic coefficient: The parameters of a fatigue curve corresponding to a pair of stresses σ_{lj} and β found without utilize the parameter ultimate stress by equation (15) and (16). The equations (15), (16), (17), and (18) utilize the physical endurance limit for limiting stress determination. The hyperbolic coefficient gradually increases when the stress level are lowered.

$$\sigma_{lj} = \frac{(\sigma_2 - \sigma_E)\sigma_1 N_2 - (\sigma_1 - \sigma_E)\sigma_2 N_1}{(\sigma_2 - \sigma_{-1})N_2 - (\sigma_1 - \sigma_{-1})N_1} \quad (15)$$

$$\beta_j = \frac{(\sigma_1 - \sigma_E)N_1}{\sigma_{lj} - \sigma_1} = \frac{(\sigma_2 - \sigma_E)N_2}{\sigma_{lj} - \sigma_2} \quad (16)$$

The above parameters can be found by equation (17) and (18) also

$$\beta_j = \frac{(\sigma_2 - \sigma_E)N_2 - (\sigma_1 - \sigma_E)N_1}{(\sigma_2 - \sigma_{-1})} \quad (17)$$

$$\sigma_{lj} = \sigma_1 + \frac{(\sigma_1 - \sigma_E)N_1}{\beta_j} = \sigma_2 + \frac{(\sigma_2 - \sigma_E)N_2}{\beta_j} \quad (18)$$

Table (8): Limiting stress and hyperbolic coefficient.

Sr. No.	Stress level number	Stress limit σ_{lj}	Co-efficient β
1	1,2	565.17	56864.44
2	1,3	544.97	83001.18
3	1,4	548.14	77415.80
4	2,3	489.94	104964.80
5	2,4	503.04	91492.08
6	3,4	961.69	32112.22

V. Comparison of calculated endurance limit error by using Gatts Equation, Exponential and hyperbolic Function: Endurance limit of AISI 1141 microalloyed steel was found by Gatts equation, exponential function and hyperbolic function. The values are tabulated in table (9).

Table (9): Calculated endurance limit, error% for microalloyed steel.

No.	Stress level number	Calculated endurance limit σ_{EC}			% error		
		Gatts Equation	Exponential Function	Hyperbolic Function	Gatts Equation	Exponential Function	Hyperbolic Function
1	1,2,3	241.27	254.65	253.98	0.53	6.10	5.83
2	1,2,4	237.77	243.96	243.64	0.93	1.65	1.51
3	1,3,4	235.93	238.68	238.52	1.7	0.54	0.62
4	2,3,4	235.66	237.90	237.78	1.8	0.87	0.92
Average		237.66	243.78	243.48	1.24	2.29	2.22

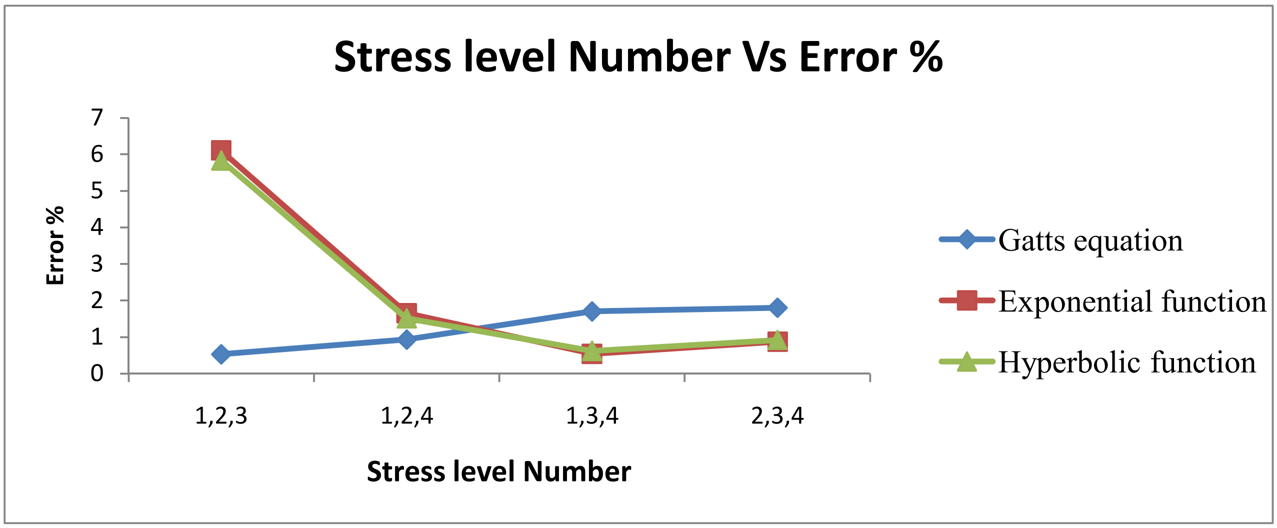

Figure (1): Stress level number Vs Calculated endurance limit error % in Gatts equation, exponential function, hyperbolic function.

The table (9) and figure (1) shows the deviation of calculated endurance limit in Gatts equation, exponential function and hyperbolic function. The hyperbolic function has the highest average error of 2.29 % the lowest deviation in error shown by Gatts equation which is 1.24. The stress combination plays vital role in endurance limit calculation. For any combination of stress, the deviation is very closer in hyperbolic and exponential function.

VI. Conclusions:

1. Gatts equation gives lower error compared to exponential function and Hyperbolic function
2. The hyperbolic function has moderate error in all stress combination.
3. The exponential function and hyperbolic function are showing very closer deviation.
4. In comparison to two stress level analysis, three stress level analysis gives accurate results in endurance limit
5. The stress combination has high power in fatigue curve description in all the 3 methods.

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