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RESEARCH ARTICLE

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## Analytical study of the optimum mass flow rate of water in a natural circulation solar collector

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**Abstract:** The work carried out in this project is based on analytical study. The performance parameters of a natural circulation solar collector have been evaluated using the empirical relations formulated and discussed in the literature. The study emphasizes on the method of evaluation of optimum mass flow rate of water in a Solar Collector for a given inlet and outlet temperature of water flowing through the collector. The mass flow rate of water in the collector has been evaluated using an iteration method. An excel spreadsheet has been prepared to calculate the mass flow rate.

**Keywords:** Solar collector, Mass flow rate, Overall heat transfer coefficient, Heat removal factor.

**1 Introduction:** Solar energy can be effectively utilized for domestic purpose by employing a solar collector. In a natural circulation solar water heater the circulation of water is directly proportional to the amount of solar radiation received [1]. H. Buchberg et. al. have stated that for an effective solar collector the distance between the tilted solar absorber and the glass cover should be within the range of 4-8 cm [2]. Bukola O. Bolaji studied the system performance of a natural circulation solar water heater and found that the performance is effected by both the flow rate and solar radiation intensity [3]. Problems on heat transfer for a flat plate type solar collector were analysed by Wieslaw Gogol et al. and certain quantities were determined that could describe the working of a flat plate solar collector better than the collector thermal efficiency [6]. Study on various types of solar energy storage systems and types of solar collector are presented with their pros and cons. For low temperature applications water is used as the storing medium [7-8]. Suresh Kumar [9] in his paper proposed empirical relation for the temperature of glass cover in a single glazed flat plate collector. Calculation of heat losses is necessary for the design of a solar collector and it was found that the maximum heat loss is from the top of the collector through the glass cover [10].

**2. Method:** The work carried out in this project is currently based on analytical study. The performance parameters of the solar water heater are evaluated using the empirical relations formulated and discussed in the literature. The methodology to calculate different parameters that are useful for the prediction of the performance of solar water heater and the heat storage system.

### 2.1 Calculation of overall heat loss coefficient of the solar collector:

$$U=U_t+ U_b+ U_s \quad (1)$$

#### 2.1.1 The top loss coefficient, $U_t$ [10]:

$$U_t^{-1} = [h_{cpg} + h_{rpg}]^{-1} + [h_w + h_{rga}]^{-1} + L_g/k_g \quad (2)$$

The above analytical equation is valid only for solar collector with single glazing and the sky temperature is taken equal to ambient temperature.

The range of conditions over which equation (2) has been developed [11] are as follows:

$$323 \text{ K} \leq T_p \leq 423 \text{ K}$$

$$260 \text{ K} \leq T_a \leq 310 \text{ K}$$

$$0.1 \leq \epsilon_p \leq 0.95$$

$$5 \text{ W m}^{-2} \text{ K}^{-1} \leq h_w \leq 45 \text{ W m}^{-2} \text{ K}^{-1}$$

$$0^\circ \leq \beta \leq 90^\circ$$

### 2.1.2 Calculation of convective heat loss coefficient, $h_{cpg}$ :

2.1.2.1 Mean temperature of air between the absorber plate and the glass cover is given by equation

$$T_{\text{mean}} = \frac{T_p + T_g}{2} \quad (3)$$

2.1.2.2 The value of product of  $Ra_L$  and  $\cos \beta$  is calculated by equation (4)

$$Ra_L \cos \beta = g \times \frac{1}{T_{\text{mean}}} \times \frac{(T_a - T_g) \times L^3}{\nu^2} \times Pr \times \cos \beta \quad (4)$$

2.1.2.3 The appropriate equation for  $Nu_L$  is then selected [3]

$$Nu_L = 1 ; Ra_L \cos \beta < 1708$$

$$Nu_L = 1 + 1.446 \{1 - (1708 / Ra_L \cos \beta)\} ; 1708 < Ra_L \cos \beta < 5900$$

$$Nu_L = 0.229 (Ra_L \cos \beta)^{0.252} ; 5900 < Ra_L \cos \beta < 9.23 \times 10^4$$

$$Nu_L = 0.157 (Ra_L \cos \beta)^{0.285} ; 9.23 \times 10^4 < Ra_L \cos \beta < 10^6 \quad (5)$$

2.1.2.4 Convective heat transfer coefficient between the plate and the cover,  $h_{cpg}$  is then given by

$$h_{cpg} = \frac{Nu_L}{L} \times k \quad (6)$$

2.1.3 Calculation of radiative heat transfer coefficient between absorber plate and glass cover,  $h_{rpg}$  [10]

$$h_{rpg} = \frac{\sigma(T_p^2 + T_g^2)(T_p + T_g)}{\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_g} - 1} \quad (7)$$

### 2.1.3 Calculation of radiative heat transfer coefficient between glass cover and surroundings of flat plate collector, $h_{rga}$ [10]

$$h_{rga} = \frac{\sigma \varepsilon_g (T_g^2 + T_a^2)(T_g + T_a)}{T_g - T_a} \quad (8)$$

### 2.1.5 Calculation of heat loss coefficient due to wind, $h_w$ [10]

$$h_w = 5.7 + (3.8 \times v) \quad (9)$$

### 2.1.6 Calculation of bottom heat loss coefficient

$$U_b = \frac{k_i}{\delta_b} \quad (10)$$

### 2.1.7 Calculation of side heat loss coefficient

$$U_s = \frac{(L_1 + L_2) L_3 k_i}{L_1 L_2 \delta_s} \quad (11)$$

## 2.2 Estimation of glass cover temperature, $T_g$ [11]:

$$T_g = T_a + h_w^{-0.38} \left( 0.567 \varepsilon_p - 0.403 + \frac{T_p}{429} \right) (T_p - T_a) \quad (12)$$

## 2.3 Calculation of solar flux absorbed by the collector:

$$S = I_b R_b (\tau \alpha)_b + \{I_d R_d + (I_b + I_d) R_r\} (\tau \alpha)_d \quad (13)$$

## 2.4 Calculation of optimum mass flow rate [4]:

$$\dot{m} = \frac{A_c F_R}{\Delta P_t} \left[ \frac{g \beta \rho_0}{C_p} \right] [S - U (t_{fi} - T_a)] \left[ \frac{1}{2} L \sin \theta + H \right] \quad (14)$$

### 2.4.1 The effective pressure difference due to buoyancy forces responsible for a closed loop cycle in natural circulation system is made up of two parts as follows:

$$\Delta P_t = \Delta P_1 + \Delta P_2 \quad (15)$$

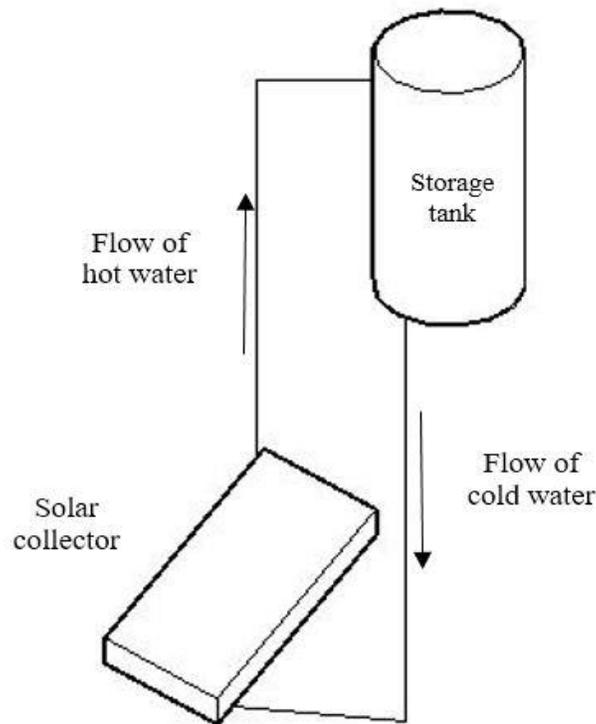


Figure (1): Setup of a natural circulation solar water heater

**2.4.2 The buoyancy pressure in the collector [4] is calculated from the equation:**

$$\Delta P_1 = g \sin \theta \int_0^{L_1} (\rho_i - \rho_x) dx \quad (16)$$

**2.4.3 The buoyancy pressure between the outlet from the collector and inlet in the storage tank [4] is calculated from the equation**

$$\Delta P_2 = (\rho_i - \rho_0) g H \quad (17)$$

**2.5 Calculation of heat removal factor,  $F_R$ :**

$$F_R = \frac{\dot{m} C_p}{U A_p} \left[ 1 - \exp \left\{ - \frac{F' U A_p}{\dot{m} C_p} \right\} \right] \quad (18)$$

**2.5.1 Collector efficiency factor,  $F'$**

$$F' = \frac{1}{W U \left[ \frac{1}{U [(W-d_0)\phi + d_0]} + \frac{\delta_a}{k_a d_0} + \frac{1}{\pi d_i h_f} \right]} \quad (19)$$

**2.5.2 Calculation of Effectiveness,  $\Phi$**

$$\Phi = \frac{\tanh\left(\frac{m(W-d_0)}{2}\right)}{\frac{m(W-d_0)}{2}} \quad (20)$$

where,

$$m = \left(\frac{U}{k_p L_p}\right)^{0.5} \quad (21)$$

## 2.6 Calculation of mean plate temperature, $T_{pm}$ :

### 2.6.1 The useful heat gain is given by

$$q_u = F_R A_c [S - U(T_{fi} - T_a)] \quad (22)$$

### 2.6.2 Heat lost is given by

$$q_l = (S \times A_c) - q_u \quad (23)$$

### 2.6.3 Plate mean temperature is given by

$$q_l = U \times A_c (T_{pm} - T_a) \quad (24)$$

## 2.7 Instantaneous Efficiency can be calculated by using the formula

$$\eta = \frac{q_u}{I_t \times A_c} \quad (25)$$

**3. Results and discussion:** The overall heat transfer coefficient has been calculated using the equations (1)-(11). The convective heat transfer coefficient,  $h_{cpg}$  has been calculated by using the correlation (5) for Nusselt number as proposed by Buchberg et al. [3]. The glass cover temperature,  $T_g$  [11] is estimated using the empirical equation (12). The calculation of  $T_g$  is done by assuming a value of the absorber plate temperature,  $T_{pm}$ . This is the first assumption in the iterative process. By using this value of  $T_g$ , the optimal mass flow rate,  $\dot{m}$ , as proposed by Bolaji B.O. [4] is calculated. The mass flow rate relation is a heat balance equation given by (14). From this equation it is evident that  $\dot{m}$  is dependent on the heat removal factor,  $F_R$  and inlet water temperature,  $t_{fi}$ . The values of  $t_{fi}$  and  $t_{fo}$  can be taken from the experimental setup and a value of  $F_R$  can be assumed. Hence from the equation (14),  $\dot{m}$  is calculated. Using this value of  $\dot{m}$ ,  $F_R$  is evaluated using the relations (18)-(21). The value of  $F_R$  thus obtained is compared with the assumed value of  $F_R$ . If both the values differ then the obtained value of  $F_R$  is put in the equation (14) and the iteration is carried on until the value of  $F_R$  matches. After this iteration is completed, the value of  $T_{pm}$  is evaluated using the relations (22)-(24) and is compared with the assumed value of  $T_{pm}$ . For any mismatch in the value of  $T_{pm}$ , the entire iteration is repeated until the assumed and the obtained values matches. The instantaneous efficiency is finally calculated using the equation (24). The calculations and iterations were done by preparing a Microsoft Excel spreadsheet.

All the calculations performed are for the location Ranchi (23°21' N, 85°19'48" E) on 1<sup>st</sup> September.

**3.1 Mass flow rate:** The variation of mass flow rate of water through the solar collector with time as obtained is shown in the figure 2. The mass flow rate as seen from the graph increases upto a peak value of 59.13 kg/hr at time 3 p.m. and then reduces as the solar radiation and hence the plate temperature decreases.

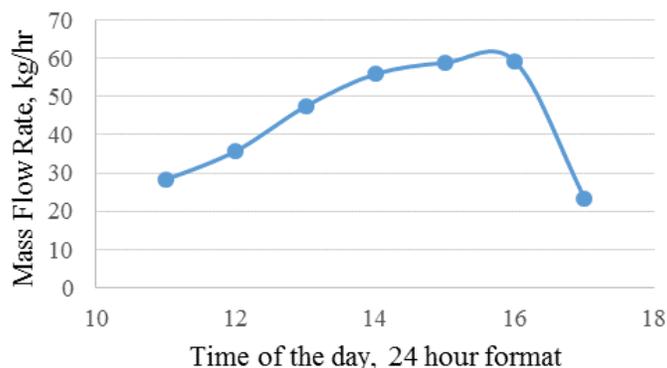


Figure (2): Mass flow rate of water through collector Vs Time

**3.2 Heat removal factor,  $F_R$ :** Figure 3 shows the variation of heat removal factor with time.  $F_R$  is also found to reach its peak value of 0.74 at time 3 p.m.

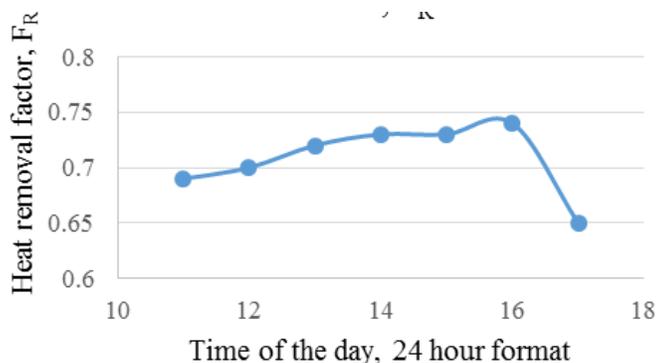


Figure (3): Heat removal factor,  $F_R$  Vs Time

**3.3 Water inlet and outlet temperature:** The variation of inlet and outlet temperature of water with time is shown in the figure 4.

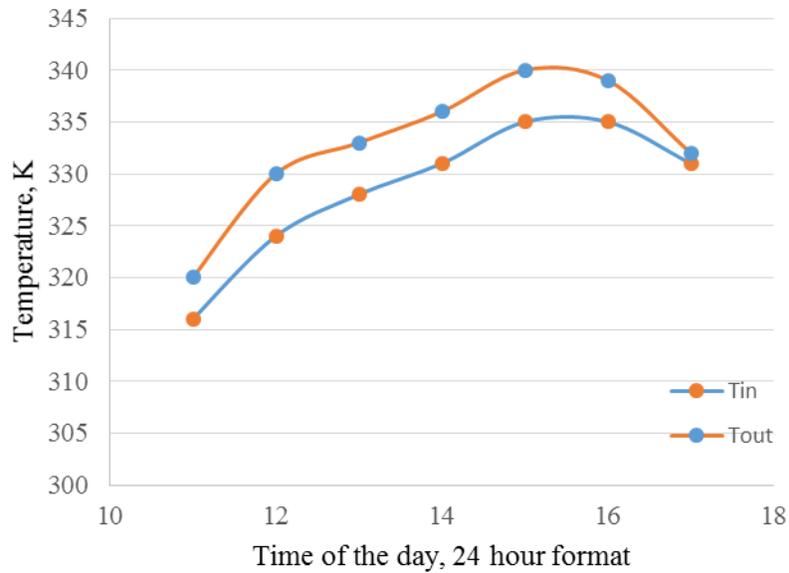


Figure (4): Water inlet outlet temperature Vs Time

**3.4 Instantaneous Efficiency,  $\eta$ :** The variation of instantaneous efficiency with time is as shown in figure 5. The maximum efficiency is achieved at 1 p.m. and its value is about 44.36 %.

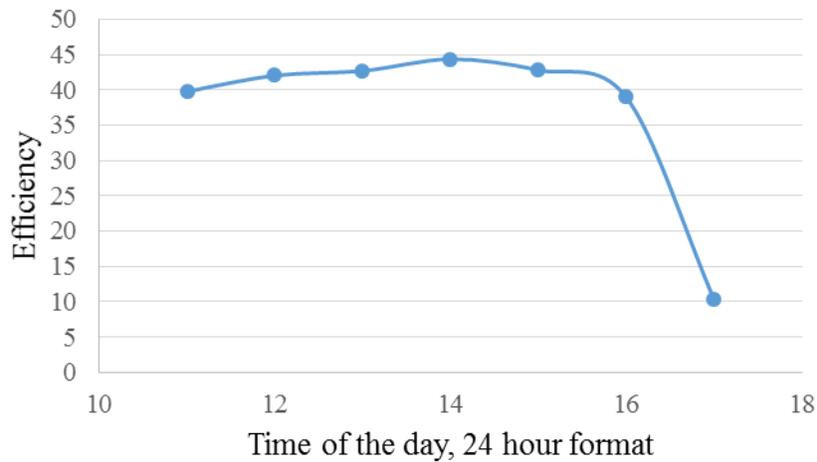


Figure (5): Efficiency Vs Time

**Nomenclature:**

- $A_p$  : Collector area,  $m^2$
- $C_p$  : Specific heat,  $J/kg-K$
- $d_i$  : Inner diameter of tube,  $m$
- $d_o$  : Outer diameter of tube,  $m$
- $F_R$  : Heat removal factor
- $F'$  : Collector efficiency factor
- $h_f$  : Heat transfer coefficient,  $W/m^2-K$
- $h_{cpg}$  : Convective coefficient between the plate and the cover,  $W/m^2-K$
- $h_{rga}$  : Radiative heat transfer coefficient between cover and surrounding,  $W/m^2-K$
- $h_{rpg}$  : Radiative heat transfer coefficient between plate and cover,  $W/m^2-K$
- $h_w$  : Heat Loss Coefficient Due To Wind,  $W/m^2-K$

$I_b$  : Solar intensity beam,  $W/m^2$   
 $I_d$  : Solar intensity diffuse,  $W/m^2$   
 $I_t$  : Flux on tilted surface,  $W/m^2$   
 $K$  : Extinction Coefficient of the cover material  
 $K$  : Thermal conductivity of air at temperature,  $W/m-K$   
 $k_i$  : Thermal conductivity of insulation,  $W/m-K$   
 $k_g$  : Thermal conductivity of glass cover,  $W/m-K$   
 $L$  : Plate to cover spacing, m  
 $L_g$  : Thickness of glass cover, m  
 $L_1$  : Length of the collector, m  
 $L_2$  : Width of the collector, m  
 $L_3$  : Height of the collector, m  
 $L_t$  : Length of tube, m  
 $\dot{m}$  : mass flow rate, kg/s  
 $Nu_L$  : Nusselt number  
 $Q_l$  : Heat lost, W  
 $Q_u$  : Useful heat gain, W  
 $Re$  : Reynolds number  
 $R_b$  : Tilt factor for beam radiation  
 $R_d$  : Tilt factor for diffuse radiation  
 $R_r$  : Tilt factor for reflected radiation  
 $S$  : Incident flux absorbed by absorber plate,  $W/m^2$   
 $T_a$  : Ambient air and sky temperature, K  
 $T_g$  : Temperature of glass cover, K  
 $T_{mean}$  : Mean temperature of air between plate and cover, K  
 $T_{pm}$  : Mean plate temperature, K  
 $t_{fi}$  : Water inlet temp, K  
 $t_{fo}$  : Water outlet temp, K  
 $U$  : Overall heat loss coefficient,  $W/m^2-K$   
 $U_t$  : Top heat loss coefficient,  $W/m^2-K$   
 $U_b$  : Bottom heat loss coefficient,  $W/m^2-K$   
 $U_s$  : Side heat loss coefficient,  $W/m^2-K$   
 $v$  : Wind speed, m/s  
 $W$  : Tube centre to centre distance, m

#### Greek symbols

$\beta$  : Collector tilt/Latitude, degree  
 $\Delta P$  : Net pressure difference due to buoyancy force,  $N/m^2$   
 $\delta$  : Declination, degree  
 $\delta_b$  : Back insulation thickness, m  
 $\delta_s$  : Side insulation thickness, m  
 $\epsilon_g$  : Glass cover emittance  
 $\epsilon_p$  : Plate emittance  
 $\eta$  : Efficiency  
 $\Theta_T$  : Angle of incidence, degree  
 $\Theta_z$  : Azimuth angle, degree  
 $\phi$  : Effectiveness  
 $\rho$  : Density of water,  $kg/m^3$   
 $\rho_d$  : Diffuse reflectance  
 $\rho_1, \rho_2$  : Reflectivity

$\sigma$  : Stefan–Boltzman constant

$\tau$  : Transmissivity

$\nu$  : Viscosity,  $m^2/s$

$\Phi$  : Latitude, degree

$\omega$  : Hour angle, degree

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