Natural convection in inclined rectangular porous enclosure with diathermal partition wall

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Abstract: Heat transfer in an unsteady natural convection within a fluid-saturated rectangular porous enclosure having vertical diathermal partition wall is studied for different inclination angles in this paper. The primary objective is to curtail the heat transfer rate across a differentially heated porous enclosure by aligning the diathermal partition vertically and to manifest the effect of inclination angle on the same. Governing equations are developed using Darcy's model and solved numerically by Successive Accelerated Replacement scheme and explicit scheme. Two cases are studied: enclosure with no partition and enclosure with vertical diathermal partition. For both the cases, fluid flow is analyzed by observing streamlines and isotherms while heat transfer is evaluated by calculating average Nusselt number. Numerical analysis is carried out for modified Rayleigh number 1000 and inclination angle 45˚, 0˚, and -45˚. It is found that employing the diathermal partition attenuates the heat transfer rate considerably. Negative inclination of enclosure showed higher attenuation of Nusselt number than enclosure with positive inclination.

Keywords: Porous media, partition, natural convection

1 Introduction: Most materials in engineering applications like insulation, cork, cement, ceramics, desalination equipment’s, rocks, wood, nuclear fuels etc. are porous in nature. The current work is based on attenuating the heat transfer rate through a fluid saturated porous medium within a square enclosure. The objective is to investigate the effect of inclination angle on heat transfer in a fluid-saturated rectangular porous enclosure with and without vertical diathermal partition wall. Comparison between both the types has been done based on this analysis.

2. Mathematical formulation: Consider an incompressible, laminar, two dimensional, unsteady natural convection flow in a rectangular porous enclosure of length $L$ and height $H$. Figure 1 shows the physical model without and with vertical partition. Initially, complete enclosure is at a lower temperature $T_c$. For $\tau > 0$, the left wall is subjected to a higher uniform temperature $T_h$, while the bottom and top wall are insulated. The outer boundaries are considered impermeable and Boussinesq approximation is applied. The viscous and radiation effects are ignored in energy conservation equation. The solid matrix of the porous media is assumed rigid; porous bed is assumed homogenous, isotropic, and saturated with incompressible fluid. Further, a Local Thermal Equilibrium is assumed to be present between porous medium and fluid. Moreover, the partition under study is assumed to be diathermal, impermeable, rigid and thin.

The conservation equations for momentum and energy equations for Darcy flow model in non-dimensional form are shown below. $X (= x/L)$, $Y (= y/L)$ are non-dimensional x and y coordinates, $\psi$ and $\theta$ are non-dimensional stream function and temperature ($T-T_c$/$T_h-T_c$), $\gamma$ is inclination angle, $Ra$ is modified Rayleigh number ($Kg$βΔTL/αν) and $\tau$ is non-dimensional time ($at/L^2$).
\[
\frac{\partial^2 \varphi}{\partial X^2} + \frac{\partial^2 \varphi}{\partial Y^2} + Ra \left[ \frac{\partial \vartheta}{\partial X} \cos \gamma - \frac{\partial \vartheta}{\partial Y} \sin \gamma \right] = 0
\]  \hspace{1cm} (1)

\[
\frac{\partial \vartheta}{\partial \tau} + \frac{\partial \vartheta}{\partial X} \frac{\partial \varphi}{\partial Y} - \frac{\partial \vartheta}{\partial Y} \frac{\partial \varphi}{\partial X} = \frac{\partial^2 \vartheta}{\partial X^2} + \frac{\partial^2 \vartheta}{\partial Y^2}
\]  \hspace{1cm} (2)

Figure (1): Physical model of enclosure with (a) no partition; (b) vertical partition.

Above equations are subjected to following initial and boundary conditions,

\[
\vartheta = 0, \quad \varphi = 0 \quad \text{at} \quad \tau = 0
\]
\[
\vartheta = 1, \quad \varphi = 0 \quad \text{at} \quad X = 0, \quad \tau > 0
\]
\[
\vartheta = 0, \quad \varphi = 0 \quad \text{at} \quad X = 1, \quad \tau = 0
\]
\[
\frac{\partial \vartheta}{\partial Y} = 0, \quad \varphi = 0 \quad \text{at} \quad Y = 0,1, \quad \tau = 0
\]  \hspace{1cm} (3)

The suitable condition at vertical partition is,

\[
\left( \frac{\partial \vartheta}{\partial X} \right)^- = \left( \frac{\partial \vartheta}{\partial X} \right)^+, \quad \varphi = 0 \quad \text{at} \quad X = 0.5
\]  \hspace{1cm} (4)

Average Nusselt number is calculated as shown below. AR is aspect ratio given by $H/L$.

\[
\bar{Nu}_{h,c} = -\frac{1}{AR} \int_{Y_0}^{Y_1} \frac{\partial \vartheta}{\partial X}(Y) dY
\]  \hspace{1cm} (5)

Equations (1 and 2) along with boundary conditions in equation (3) and suitable condition at partition in equation (4) were numerically solved using Successive Acceleration Replacement (SAR) scheme. For discretization of equations, Finite Difference Method (FDM) with second-order accuracy in central difference was used while a second order forward and backward difference was used at the wall boundaries. Unsteady term was dealt using explicit scheme.
The stream function and temperature were solved using this technique for all grid points until the convergence was achieved.

3. Results and discussion: The effect of inclination angle ($\gamma$) and partition orientation was studied by noting the transient effects of streamlines and isotherms for modified $Ra = 1000$ for $\gamma = 45^\circ$, $0^\circ$ and $-45^\circ$. A numerical code was developed to solve the algebraic equations resulted from FDM scheme. In view of both computational cost and precision, the results obtained with the 61 x 61 mesh were considered acceptable. Since the current problem has aspect ratio 2, instead of mesh size the grid size was maintained constant making the mesh size as 61 x ($AR \times 61$). For validation, the values of average Nusselt number were compared with similar works in literature for porous enclosure with no partition having $AR = 1$ and $Ra = 10, 100$ and 1000 which are shown in Table (1).

<table>
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<tr>
<th>Authors/Ra</th>
<th>10</th>
<th>100</th>
<th>1000</th>
</tr>
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<tbody>
<tr>
<td>BEJAN [1]</td>
<td>-</td>
<td>4.2</td>
<td>15.8</td>
</tr>
<tr>
<td>PRESENT STUDY</td>
<td>1.081</td>
<td>3.309</td>
<td>15.723</td>
</tr>
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Based on above validation, it can be concluded that the current work shows a good agreement with the works in literature and thus, present computational code can be used with greater assurance to study the problem stated in this paper. Figure (2) shows streamlines and isotherms for enclosure with no partition (Figure 2a) and vertical partition (Figure 2b) for modified Rayleigh number 1000 and inclination angle $45^\circ$, $0^\circ$ and $-45^\circ$ at steady state. When $\gamma = 45^\circ$, hot wall is below and cold wall is above. This arrangement aids the convection flow as warm air always tends to move upwards i.e., towards the cold wall. When $\gamma = -45^\circ$, hot wall is above and cold wall is below. This causes accumulation of warm air towards the hot wall and together this accumulation reaches the cold wall as a whole bunch which is seen clearly in streamline and isotherm contours for $\gamma = -45^\circ$ in both the cases. Due to vertical partition, convection in second block does not start unless and until the convection in first block is significantly developed.

Figure (3a) shows the effect of inclination angle on stream function at steady state. As $\gamma$ increases, the maximum absolute stream function value also increases. This suggests that the strength of convection increases as $\gamma$ varies from $-45^\circ$ to $45^\circ$ which in turn increases the average Nusselt number value as seen clearly in Figure (3b). It is clear that Nusselt number shows a significant drop when $\gamma$ decreases from $0^\circ$ to $-45^\circ$ for both the cases with vertical partitioned enclosure having the least value of $Nu$ for entire range of $\gamma$. 


Figure (2): Streamlines (up) & isotherms (down) at steady state for enclosure with (a) no partition; (b) vertical partition.
4. Conclusions: Numerical analysis of rectangular porous enclosure with and without vertical partition for varying inclination angle has been carried out in this paper. It is found that employing vertical diathermal partition decreases the Nusselt number considerably. Nusselt number also shows a significant drop when $\gamma$ decreases from $0^\circ$ to $-45^\circ$ for both the cases. For $0^\circ < \gamma < 45^\circ$, Nusselt number increases up to roughly $25^\circ$ after which it starts decreasing. Further, vertically partitioned enclosure yields the least value of $\text{Nu}$ for entire range of $\gamma$. 

Figure (3): Variation of: (a) absolute maximum stream function and (b) average Nusselt number with inclination angle at steady state.
References:

